RMO 2017

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- Let AOB be a given angle less than 180^o and let P be an interior point of the angular region determined by $\angle AOB$ .Show, with proof, how to construct, using only ruler and compass, a line segment CD passing through P such that C lies on the ray OA and D lies on the ray OB and CP:PD = 1:2.

- Show that the equation

  $$a^3+(a+1)^3+(a+2)^3+(a+3)^3+(a+4)^3+(a+5)^3+(a+6)^3 = b^4+(b+1)^4$$

  has no solutions in integer a, b.

- Let $P(x) = x^2 + \frac{1}{2}x + b$ and $Q(x) = x^2 + cx + d$ be two polynomials with real coefficients such that P(x) Q(x) = Q(P(x)) for all real x. Find all real roots of P(Q(x)) = 0

- Consider n^2 unit squares in the xy-plane centered at the point (i, j) with integer coordinates, $1 \leq i \leq n$, $1 \leq j \leq n$. It is required to color each unit square in such a way that whenever $1 \leq i < j \leq n$ and $1 \leq k < l \leq n$ the thre squares with centers at (i, k), (j, k) , (j, l) have distinct colours. What is the least possible colours needed?

- Let Ω be a circle with a chord AB which is not a diameter. Let Γ_1 be a circle on one side of AB such that it is tangent to AB at C and internally tangent to Ω at D. Likewise let Γ_2 be a circle on the other side of AB such that it is tangent to AB at E and internally tangent to Ω at F. Suppose the line DC intersects Ω at $X \neq D$ and the line FE intersects Ω at $Y \neq F$. Prove that XY is a diameter of Ω

- Let x, y, z be real numbers, each greater than 1. Prove that

  $$\frac{x+1}{y+1} + \frac{y+1}{z+1} + \frac{z+1}{x+1} \leq \frac{x-1}{y-1} + \frac{y-1}{z-1} + \frac{z-1}{x-1}$$