1. The number of subsets of $\{1, 2, 3, \ldots, 10\}$ having an odd number of elements is

(A) 1024 (B) 512 (C) 256 (D) 50.

2. For the function on the real line $\mathbb{R}$ given by $f(x) = |x| + |x+1| + e^x$, which of the following is true ?

$f(x) = \begin{cases} 2x+1+e^x \\ 1+e^x \\ -2x-1+e^x \end{cases}$ at 0 $2+e^x$, at (-1) e^x

$x \geq 0$ $f(x)$ $[-1, 0)$ e^x $-2+e^x$

(A) It is differentiable everywhere.

(B) It is differentiable everywhere except at $x = 0$ and $x = -1$.

(C) It is differentiable everywhere except at $x = 1/2$.

(D) It is differentiable everywhere except at $x = -1/2$.

3. If f, g are real-valued differentiable functions on the real line $\mathbb{R}$ such that $f(g(x)) = x$ and $f'(x) = 1 + (f(x))^2$, then $g'(x)$ equals

$f'(g(x)) \cdot g'(x) = 1$

$f'(x) = 1 + f^2(x)$

$f'(g(x)) = 1 + f^2(g(x))$

$= 1 + x^2$

$g'(x) = \frac{1}{1+x^2}$

(A) $\dfrac{1}{1+x^2}$ (B) $1+x^2$ (C) $\dfrac{1}{1+x^4}$ (D) $1+x^4$.

4. The number of real solutions of $e^x = \sin(x)$ is

(A) 0 (B) 1 (C) 2 (D) infinite.

5. What is the limit of $\displaystyle\sum_{k=1}^{n} \frac{e^{-k/n}}{n}$ as n tends to ∞ ?

$\int_0^1 e^{-y}\,dy = -\int_0^{-1} e^{y}\,dy$

$1 - e^{-1}$

(A) The limit does not exist.

(B) ∞

(C) $1 - e^{-1}$

(D) $e^{-0.5}$

$$\frac{64!}{32!(2!)^{32}}$$

6. A group of 64 players in a chess tournament needs to be divided into 32 groups of 2 players each. In how many ways can this be done ?

(A) $\dfrac{64!}{32!2^{32}}$ (B) $\dbinom{64}{2}\dbinom{62}{2}\cdots\dbinom{4}{2}\dbinom{2}{2}$

(C) $\dfrac{64!}{32!32!}$ (D) $\dfrac{64!}{2^{64}}$

7. The integral part of $\displaystyle\sum_{n=2}^{9999} \frac{1}{\sqrt{n}}$ equals

(A) 196 (B) 197 (C) 198 (D) 199.

8. Let a_n be the number of subsets of $\{1, 2, \ldots, n\}$ that do not contain any two consecutive numbers. Then

(A) $a_n = a_{n-1} + a_{n-2}$ (B) $a_n = 2a_{n-1}$

(C) $a_n = a_{n-1} - a_{n-2}$ (D) $a_n = a_{n-1} + 2a_{n-2}$.

9. There are 128 numbers $1, 2, \ldots, 128$ which are arranged in a circular pattern in clockwise order. We start deleting numbers from this set in a clockwise fashion as follows. First delete the number 2, then skip the next available number (which is 3) and delete 4. Continue in this manner, that is, after deleting a number, skip the next available number clockwise and delete the number available after that, till only one number remains. What is the last number left ?

(A) 1 (B) 63 (C) 127 (D) None of the above.

10. Let z and w be complex numbers lying on the circles of radii 2 and 3 respectively, with centre $(0, 0)$. If the angle between the corresponding vectors is 60 degrees, then the value of $|z + w|/|z - w|$ is:

(A) $\dfrac{\sqrt{19}}{\sqrt{7}}$ (B) $\dfrac{\sqrt{7}}{\sqrt{19}}$ (C) $\dfrac{\sqrt{12}}{\sqrt{7}}$ (D) $\dfrac{\sqrt{7}}{\sqrt{12}}$.

11. Two vertices of a square lie on a circle of radius r and the other two vertices lie on a tangent to this circle. Then the length of the side of the square is

(A) $\frac{3r}{2}$ (B) $\frac{4r}{3}$ (C) $\frac{6r}{5}$ (D) $\frac{8r}{5}$.

12. For a real number x, let $[x]$ denote the greatest integer less than or equal to x. Then the number of real solutions of $\left|2x - [x]\right| = 4$ is

(A) 4 (B) 3 (C) 2 (D) 1.

13. Let f, g be differentiable functions on the real line $\mathbb{R}$ with $f(0) > g(0)$. Assume that the set $M = \{t \in \mathbb{R} \mid f(t) = g(t)\}$ is non-empty and that $f'(t) \geq g'(t)$ for all $t \in M$. Then which of the following is necessarily true ?

(A) If $t \in M$, then $t < 0$.

(B) For any $t \in M$, $f'(t) > g'(t)$.

(C) For any $t \notin M$, $f(t) > g(t)$.

(D) None of the above.

14. Consider the sequence $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, \ldots$ obtained by writing one 1, two 2's, three 3's and so on. What is the 2020^{th} term in the sequence ?

(A) 62 (B) 63 (C) 64 (D) 65

15. Let $A = \{x_1, x_2, \ldots, x_{50}\}$ and $B = \{y_1, y_2, \ldots, y_{20}\}$ be two sets of real numbers. What is the total number of functions $f\colon A \to B$ such that f is onto and $f(x_1) \leq f(x_2) \leq \cdots \leq f(x_{50})$?

(A) $\binom{49}{19}$ (B) $\binom{49}{20}$ (C) $\binom{50}{19}$ (D) $\binom{50}{20}$

16. The number of complex roots of the polynomial $z^5 - z^4 - 1$ which have modulus 1 is

$z = \cos\theta + i\sin\theta$

(A) 0 (B) 1 (C) 2 (D) more than 2.

$\cos 5\theta = 1 + \cos 4\theta$

$-2\sin\left(\frac{9\theta}{2}\right)\sin\left(\frac{\theta}{2}\right) = 1$

$\cos 5\theta + i\sin 5\theta - \cos 4\theta - i\sin 4\theta - 1 = 0$

$2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{9\theta}{2}\right) = 0$ & $\sin 5\theta \cdot \sin 4\theta = 0$

17. The number of real roots of the polynomial

$\cos\left(\frac{9\theta}{2}\right) = 0$

$\Rightarrow \sin\frac{\theta}{2} = \pm\frac{1}{2}$

$\frac{\theta}{2} = n\pi \pm \pi/6$

$\frac{9\theta}{2} = 9n\pi \pm \frac{3\pi}{2}$

$\cos\left(\frac{9\theta}{2}\right) = \cos\left(n\pi \pm \frac{\pi}{2}\right)$

$$p(x) = (x^{2020} + 2020x^2 + 2020)\underbrace{(x^3 - 2020)}_{1}\underbrace{(x^2 - 2020)}_{2}$$

is

(A) 2 ~~(B) 3~~ (C) 2023 (D) 2025.

18. Which of the following is the sum of an infinite geometric sequence whose terms come from the set $\{1, \frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{2^n}, \ldots\}$?

(A) $\frac{1}{5}$ ~~(B)~~ $\frac{1}{7}$ (C) $\frac{1}{9}$ (D) $\frac{1}{11}$

$\frac{1/2^a}{1 - 1/2^a} = \frac{1}{2^a - 1}$

$= 1 + \frac{1}{2^a - 1}$ $1 + \frac{1}{2^a} + \frac{1}{2^{2a}} + \cdots = \frac{a}{1-r} = \frac{1}{1 - (1/2^a)} = \frac{2^a}{2^a - 1}$

19. If a, b, c are distinct odd natural numbers, then the number of rational roots of the polynomial $ax^2 + bx + c$

(A) must be 0.

(B) must be 1.

(C) must be 2.

(D) cannot be determined from the given data.

20. Let A, B, C be finite subsets of the plane such that $A \cap B$, $B \cap C$ and $C \cap A$ are all empty. Let $S = A \cup B \cup C$. Assume that no three points of S are collinear and also assume that each of A, B and C has at least 3 points. Which of the following statements is always true ?

(A) There exists a triangle having a vertex from each of A, B, C that does not contain any point of S in its interior.

(B) Any triangle having a vertex from each of A, B, C must contain a point of S in its interior.

(C) There exists a triangle having a vertex from each of A, B, C that contains all the remaining points of S in its interior.

(D) There exist 2 triangles, both having a vertex from each of A, B, C such that the two triangles do not intersect.

21. Shubhaangi thinks she may be allergic to Bengal gram and takes a test that is known to give the following results:

• For people who really do have the allergy, the test says "Yes" 90% of the time.

• For people who do not have the allergy, the test says "Yes" 15% of the time.

If 2% of the population has the allergy and Shubhaangi's test says "Yes", then the chances that Shubhaangi does really have the allergy are

$\frac{x}{x+y} = \frac{2}{100} = \frac{1}{50}$

$x = 1$, $y = 49$

(A) 1/9

(B) 6/55

(C) 1/11

(D) cannot be determined from the given data.

$\frac{\frac{90}{100}(x)}{\frac{90}{100}(x) + \frac{15}{100}(y)}$ $\quad \frac{90}{90 + 15(49)} = \frac{6}{55}$

22. If $\sin(\tan^{-1}(x)) = \cot(\sin^{-1}(\sqrt{\frac{13}{17}}))$ then x is

$\sin\theta = \sqrt{\frac{13}{17}}$

$\cos\theta = \sqrt{1 - \frac{13}{17}} = \frac{2}{\sqrt{17}}$

(A) $\frac{4}{17}$ (B) $\frac{2}{3}$ (C) $\sqrt{\frac{17^2-13^2}{17^2+13^2}}$ (D) $\sqrt{\frac{17^2-13^2}{17\times13}}$.

$\cot\theta = \frac{2}{\sqrt{13}}$

$289 + 169 = 358$ $\quad \frac{17^2-13^2}{17^2+13^2} = \frac{4(30)}{358}$

$\alpha = \tan^{-1}(x)$ $\quad \sin(\alpha) = \frac{2}{\sqrt{13}} \Rightarrow \cos\alpha = \frac{3}{\sqrt{13}}$ $\quad \tan\alpha = \frac{2}{3}$

$E \to 6! = 720$

5

EEMPRTU

23. If the word PERMUTE is permuted in all possible ways and the different resulting words are written down in alphabetical order (also known as dictionary order), irrespective of whether the word has meaning or not, then the 720th word would be:

(A) EEMPRTU (B) EUTRPME (C) UTRPMEE (D) MEET-PUR.

590 240

A B C D

24. The points $(4, 7, -1)$, $(1, 2, -1)$, $(-1, -2, -1)$ and $(2, 3, -1)$ in $\mathbb{R}^3$ are the vertices of a

350 240 350

110

(A) rectangle which is not a square.

$AD = BC = \sqrt{20}$

$AB = CD = \sqrt{34}$

$AC =$

(B) rhombus.

(C) parallelogram which is not a rectangle.

(D) trapezium which is not a parallelogram.

25. Let $f(x), g(x)$ be functions on the real line $\mathbb{R}$ such that both $f(x)+g(x)$ and $f(x)g(x)$ are differentiable. Which of the following is FALSE ?

(A) $f(x)^2 + g(x)^2$ is necessarily differentiable. True

(B) $f(x)$ is differentiable if and only if $g(x)$ is differentiable. True.

(C) $f(x)$ and $g(x)$ are necessarily continuous.

(D) If $f(x) > g(x)$ for all $x \in \mathbb{R}$, then $f(x)$ is differentiable.

26. Let S be the set consisting of all those real numbers that can be written as $p - 2a$ where p and a are the perimeter and area of a right-angled triangle having base length 1. Then S is

(A) $(2, \infty)$ (B) $(1, \infty)$ (C) $(0, \infty)$ (D) the real line $\mathbb{R}$.

$p = 1 + x + \sqrt{x^2+1}$

$S = 1 + \sqrt{x^2+1}$

$a = \frac{1}{2}x$

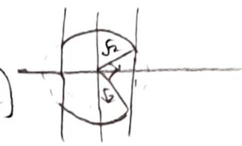
27. Let $S = \{1, 2, \ldots, n\}$. For any non-empty subset A of S, let $l(A)$ denote the largest number in A. If $f(n) = \sum_{A \subseteq S} l(A)$, that is, $f(n)$ is the sum of the numbers $l(A)$ while A ranges over all the nonempty subsets of S, then $f(n)$ is

(A) $2^n(n+1)$ (B) $2^n(n+1) - 1$

(C) $2^n(n-1)$ (D) $2^n(n-1) + 1$.

28. The area of the region in the plane $\mathbb{R}^2$ given by points (x, y) satisfying $|y| \leq 1$ and $x^2 + y^2 \leq 2$ is

(A) $\pi + 1$ (B) $2\pi - 2$ (C) $\pi + 2$ (D) $2\pi - 1$.



$\pi(2) - 2\left[\frac{1}{4}\pi(2)\right] + 2\left(\frac{1}{2} \times \sqrt{2} \times \sqrt{2}\right)$

$\pi + 2$

29. Let n be a positive integer and $t \in (0, 1)$. Then $\sum_{r=0}^{n} r \binom{n}{r} t^r (1-t)^{n-r}$ equals

(A) nt (B) $(n-1)(1-t)$ (C) $nt + (n-1)(1-t)$ (D) $(n^2 - 2n + 2)t$.

30. For any real number x, let $[x]$ be the greatest integer m such that $m \leq x$. Then the number of points of discontinuity of the function $g(x) = [x^2 - 2]$ on the interval $(-3, 3)$ is

(A) 5 (B) 9 (C) 13 (D) 16.