STUDY MATERIAL FOR AMC 8 AND MATHCOUNTS

Reading Note

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Day - 2

Topic - Geometry
Similarity and the Pythagorean Theorem

0.1 Lesson Overview

The Pythagorean theorem is one of the most fundamental results in all of Mathematics. Its effects are felt far beyond the realm of elementary Geometry, mostly because of its application to calculating distances. The distance formula in Coordinate Geometry is nothing but a reformulation of this result, and consequently it is used whenever we calculate distances in euclidean spaces.

In today's lesson, we shall see its proof and some applications. Before we prove it, we shall discuss the concept of similar triangles.

0.1.1 Similar triangles

Two triangles are said to be *similar* if they have the same angles. Let ΔABC and $\Delta A'B'C'$ be triangles such that $\angle A = \angle A', \angle B = \angle B'$ and $\angle C = \angle C'$. Symbolically, we denote this similarity as $\Delta ABC ||| \Delta A'B'C'$.

It can be shown that, in this case the corresponding sides are proportional. In other words,

$$|AB| : |BC| : |CA| = |A'B'| : |B'C'| : |C'A'|.$$

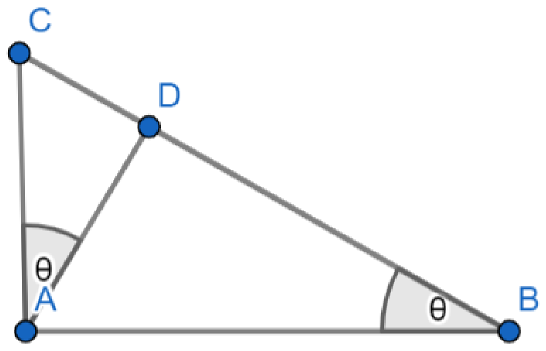
Think! Can you show that the angles being equal is equivalent to the sides being proportional?

Think! Can you prove that Area(ΔABC) : Area($\Delta A'B'C'$) $= |BC|^2 : |B'C'|^2$?

0.1.2 The Pythagorean Theorem

Theorem 0.1.1. *If ΔABC is a right angled triangle with BC as the hypotenuse (side opposite to right angle), then the following relation between side lengths holds,*

$$|AB|^2 + |AC|^2 = |BC|^2.$$



Proof. Let D be the foot of the perpendicular from A to BC. Note that $\angle CAD = \angle CBA = \angle ABD$, so the triangles $\Delta ACD, \Delta ADB$ and ΔABC are all similar (as they are all right-angled triangles).

By looking at the corresponding angles of these triangles, we may write

$$\frac{|CD|}{|CA|} = \frac{|CA|}{|BC|}$$

and

$$\frac{|BD|}{|AB|} = \frac{|AB|}{|BC|}.$$

Hence

$$|AB|^2 + |AC|^2 = |BC|(|BD| + |DC|) = |BC|^2.$$

□

Think! Suppose that BC is the largest side of ΔABC and the corresponding angle $\angle A$ is obtuse. Which one is larger, $|AB|^2 + |AC|^2$ or $|BC|^2$? Why?