



CHEENTA ACADEMY

Mahalanobis Olympiad Demo Problems

Mahalanobis Olympiad

Cheenta Academy

High School Mathematics

Problem 1

For some particular value of N , when $(a + b + c + d + 1)^N$ is expanded and like terms are combined, the resulting expression contains exactly 1001 terms that include all four variables a, b, c , and d , each to some positive power. What is N ?

Calculus and Linear Algebra

Problem 1

Describe the set of values of a for which the curves $x^2 + y^2 = a^2$ and $y = x^2 - a$ in the real xy -plane intersect at exactly 3 points?

Probability

Problem 1

Suppose $x_1, x_2, \dots, x_n$ constitute a random sample from a Bernoulli population with parameter p . Let

$$z_i = \begin{cases} 1 & \text{if } x_i \leq x_0 \\ 0 & \text{otherwise} \end{cases}$$

for $i = 1, 2, \dots, n$.

A) Find the probability distribution of z_i . Find $E(z_i)$, $V(z_i)$ and $\text{Cov}(z_i, z_j)$, $i \neq j$.

B) Let $y = \sum_{i=1}^n z_i$. Find the probability distribution of y .

Statistics

Problem 1

Suppose we have n observations $x_1, x_2, \dots, x_n$.

Let

$$\bar{x}_i = \left(\sum_{j=1}^i x_j \right) / i, \quad i = 1, 2, \dots, n$$

and

$$s^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x}_n)^2$$

.

Show that

$$ns^2 = \sum_{i=2}^n \frac{i}{i-1} (x_i - \bar{x}_i)^2$$

.

Problem 2

Let $x_1, x_2, \dots, x_n$ be a random sample from the rectangular population with density

$$f(x) = \begin{cases} \frac{1}{\theta}, & 0 < x < \theta \\ 0, & \text{otherwise} \end{cases}$$

Consider the critical region $x_{(n)} > 0.8$ for testing the hypothesis $H_0 : \theta = 1$, where $x_{(n)}$ is the largest of $x_1, x_2, \dots, x_n$. What is the associated probability of type I error and what is the power function?