Cheenta

Euleriam Graph

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Content

Lesson Overview

In this lesson we will discuss about Euleriam Graph, Planner graph and Euler's theorem on Planner Graph.

Eulerian Graph

An Eulerian path is a path that uses every edge of a graph exactly once. An Euler path starts and ends at different vertices.

A circuit is called an Eulerian circuit if it visits all the edges of the graph exactly ones. It begins and ends on the same vertices.

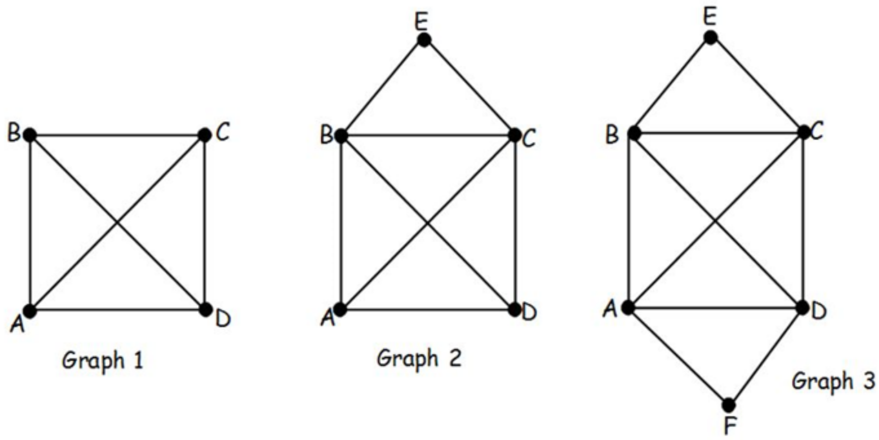
A graph containing an Eulerian circuit is called an Eulerian Graph.

Note

(i) Every Eulerian Graph is a connected graph.

(ii) A connected graph is Eulerian if the degree of all its vertices is even.

Example



Graph 1 and Graph 2 are not Eulerian Graph because in Graph 1 all the vertices are of odd degree and in graph 2 the vertices A and D are of odd degree.

In graph 3 there is an Eulerian circuit

A F D B C A D C E B A

So this is an Eulerian Graph.

Planner Graph

A graph that can be drawn in such a way that its edge does not intersect each other except at their endpoints is called a planner graph.

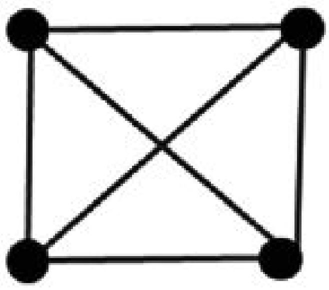


Figure 1

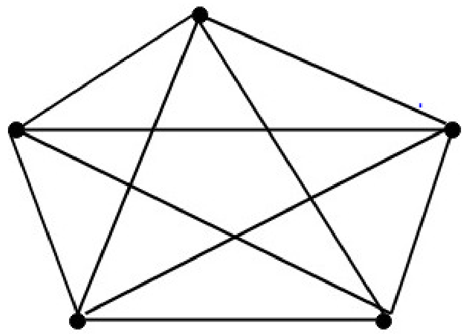


Figure 2

The graph shown in Figure 1 is planner , while the graph shown in Figure 2 is not planner.

Euler's Theorem

If a Graph is depicted properly , then it divides the graph into several regions called faces. Let the number of faces be F, number of vertices be V and the number of edges be E.

Then **Euler's Theorem** states that the quantity $V - E + F = 2$ always holds true for a planner graph.

Problems

Problem 1: There are 7 lakes in Lakeland. They are connected by 10 canals so that one can swim through the canals from any lake to any other. How many islands are there in Lakeland?

Solution : We will consider the points and the vertices of the square as the vertices, and the segments and the sides of the square as the edges of a planner graph. For each region (among those into which the graph divides the plane) we calculate the number of edges on its border. Then we add up all these numbers. Since any edge separates exactly two different faces from one another, the total must be simply double the number of edges. Since all the faces are triangles, except for the outer one, which is surrounded by four edges, we get $3(F-1)+4=2E$; that is, $E=3(F-1)/2+2$. Since the number of vertices equals 24, using Euler's formula, we have

$$24-\left(\frac{3(F-1)}{2}+2\right)+F=2$$

This gives $F=43$ (counting the "outside face"). So, the number of triangles our square is divided into is 42 .

Problem 2: Prove that for a planner graph $2E \geq 3F$.

Solution : Each face must be surrounded by at least 3 edges. Let B be the total number of boundaries around all the faces in the graph. Thus we have that $3F \leq B$. But also $B = 2E$, since each edge is used as a boundary exactly twice. Putting this together we get $2E \geq 3F$.

Problem 3: Prove that for a planar connected graph E $\leq 3V - 6$.

Solution : From the previous problem $2E \geq 3F$.

Substituting this into Euler's formula $V - E + F = 2$

we have $V - E + 2E/3 \geq 2$.

That gives $E \leq 3V - 6$, as required.

Note

Combining $3V - 6 \geq E$ and $2E \geq 3F$

we get $2V - 4 \geq F$

So the number of faces of a Connected Planner graph cannot exceed $2V - 4$.

Problem 4

Prove that the graph with 5 vertices, each of which is connected by an edge to every other, is not planar.

Problem 5

Is it possible to build three houses and three wells, then connect each house with each well by nine paths, no two of which intersect except at their end-points?