

Mathematical Circles

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Introduction

My name is Sharanyaa. I study in grade 6. As a student of Cheenta, on 25th June, 2023 I led a math circle session with two grade 6 students from Swapnopuran Welfare Society (SWS) school.

Tools Used

We met virtually through Google Meet. I used Microsoft PowerPoint, as the virtual board.

Problem 1

Alex has 2 ropes of lengths 1m and 2m. He cuts both ropes so that all pieces are of equal length. Which of the following number of pieces can he not obtain?

a) 6 b) 8 c) 9 d) 12 e) 15

When I started this problem, the class was relatively quiet and none of the participants were willing to share their views. One of the participants facing internet issues had to drop out of the call and I was unable to learn her views. However, the other participant thought the answer was 15. Even though the answer was wrong, I was glad that she answered. After all, a wrong answer is better than no answer!

(Considering 1m rope as r and 2m rope as s)

I told her that if r was divided into 2 pieces measuring 50 cm each, then s would be divided into 4 pieces. Similarly, I asked her to find the number of pieces s would be divided into if r was divided into 4 pieces. She realized that if r was divided into 4 pieces, each piece would measure 25 cm. So, if we divide 200 cm by 25 cm we will see that s was divided into 8 pieces. After solving this, she realized that the number of pieces rope r was divided into would be half of the number of pieces rope s was divided into. Thus, we realized that:

Number of pieces from <i>r</i>	Number of pieces from s	Total no. of pieces
1	2	3
2	4	6
3	6	9
4	8	12
5	10	15

We realized that 8 is somewhere in between 6 and 9 in the column for total pieces. So, we decided to find out how 8 could be expressed as the sum of two numbers:

1 + 7 = 8
2+6=8
3 + 5 = 8
4 + 4 = 8

As you can see, 1 is not the half of 7. Similarly, 2 is not half of 6, 3 is not half of 5 and 4 is equal to 4. Thus, we realized that we could not get 8 pieces of equal length from the two ropes and the fact that our answer was b) 8.

Cosmic routes are established between the nine planets of the solar system. Rockets travel along the following routes: Earth-Mercury, Pluto-Venus, Earth-Pluto, Pluto-Mercury, Mercury-Venus, Uranus-Neptune, Neptune-Saturn, Saturn-Jupiter, Jupiter-Mars and Mars-Uranus. Can a traveler get from Earth to Mars?

Understanding this problem is very difficult. However, the solution is easy. I had to explain this problem twice. I was pleasantly surprised to find the solution ready for me. The participant explained that the Earth was only connected with Mercury, Venus and Pluto while Mars was connected with Jupiter, Saturn, Uranus and Neptune. This can be explained with a simple diagram:

As shown by the pink lines, Mercury, Venus, Earth and Pluto are connected. As shown with yellow lines, Mars, Jupiter, Saturn, Uranus and Neptune are connected.

Thus, the answer is a traveler can't travel from Earth to Mars

This problem introduced the participants to the basic concept of graph.



Lisa, Mina, Kianna and Claire have pets: a dog, a cat, a goldfish and a parakeet. Each girl owns exactly one pet and we know the following facts:

- \rightarrow Lisa's pet has black fur
- → Kianna's pet cannot swim
- $\rightarrow~$ Lisa and Kianna are not fond of cats
- \rightarrow Claire's pet has four paws

Name the owners of each of the animals.

After explaining the problem once, I was once again astonished to learn that one of the participants seemed to find the answer in a few minutes! She gave me the correct answer:

Lisa – Dog		
Kianna – Parakeet		
Claire – Cat		
Mina – Goldfish		

She explained that

- Only cats and dogs can have black fur. Since Lisa did not like cats, she could have only a dog.
- Cats and birds can't swim. Since Kianna's pet can only be a cat or a parakeet and she doesn't like cats, her pet is a parakeet.
- Only a dog or a cat can have four paws and Lisa had a dog, so Claire had a cat.
- This meant Mina had a goldfish.

A bag contains beads of two colours: black and white. What is the smallest number of beads which must be drawn from the bag, without looking, so that among these beads there are 2 of the same colour?

a) 2 b) 3 c) 4 d) 5 e) 6

I explained this problem twice. However, the participant was unable to solve them. After she asked me for the solution, I started explaining.

Let us consider there is a bag with many beads of the colours black and white. Now let us say that 2 is the answer. If we take 2 beads out of the bag (without looking), following are the possibilities:

	Two black beads	Drawn two beads of same colour
Best Case		
	Two white beads	Drawn two beads of same colour
Worst Case	One white and one black bead	Two beads have different colours

Now let us imagine that we draw the 3rd bead out of the bag. Even if the first two beads are of the colours black and white, we will get at least two beads of the same colour as the third bead would be black or white.

	Two black beads	Drawn two beads of same colour
Best Case		
	Two white beads	Drawn two beads of same colour
Worst Case	One black bead, one white bead and one black or white bead	Out of 3 beads 2 have same colour

So the answer is we need to draw a minimum of 3 beads to satisfy the above condition.

After describing the solution, when I asked her to explain it, she managed to do so. However, she thought the number of beads being removed was the total number of beads in the bag. I cleared this doubt and finished this problem.

I have 2 pairs of blue socks and 3 pairs of green socks. What is the minimum number of socks I have to remove to get 1 pair of matching socks?

This problem was given as homework. What do you think the answer is?

Experience

I am very happy that I got to lead this session and the participant excelled in the problems. This session gave both of us the opportunity to think and discuss about few problems. I wish I could have discussed with the other participant. Unfortunately, she had to drop because of internet connectivity issues. We have shared the recording of the session with her. All in all, it was a wonderful session. I hope to meet with them soon.

